

# Experimental investigation on the MRTLCD for vibration control of slender structures

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## Abstract

An experimental investigation into the effectiveness of Magnetorheological Tuned Liquid Column Dampers (MRTLCDs) for the reduction of structural accelerations in a cantilevered (SDOF) structure subjected to sinusoidal and random excitation is carried out. A tall, slender annular pipe is used to model the tubular tower of a flexible structure and a pre-determined steel weight was placed at the top of the tube in order to deliver the required fundamental natural frequency. An MR fluid, which possesses a relatively low viscosity and density, is used as the residing liquid in the MRTLCD. The response of the structure-MRTLCD system to sinusoidal and random excitations with various magnetic flux densities applied transversely to the fluid flow in the MRTLCD is investigated. The advantages of MRTLCDs and the practicality of utilising MR fluids in TLCDS are examined and concluded upon.

## 1 Introduction

The abatement of vibrations induced within long-period civil engineering structures using Tuned Liquid Column Dampers (TLCDS) have been studied extensively ever since an initial study into the subject was made in 1989 [1]. TLCDS are attractive for vibration control as they are consistent over a wide range of excitation levels, prove highly efficient in respect to volumetric efficiency compared to other liquid dampers and also because TLCDS are self-contained, with no auxiliary equipment, personnel or power required to operate and maintain it. The use of TLCDS in mitigating vibrations within civil engineering structures has also been extensively studied [2-7]. The tuning ratio of the MRTLCD is defined as the ratio of the natural frequency of the MRTLCD to the natural frequency of the structure. The influence of the tuning ratio on the performance of the TLCDS has been studied [8-10] and it was found that it has a large impact on the performance of a TLCDS. The coefficient of head loss in the TLCDS may be changed by varying the orifice opening within the horizontal pipe in the TLCDS. Appropriate changes in the orifice opening optimize the damping effect of the TLCDS in relation to varying excitations and structural natural frequencies. An active tuned liquid column damper (ATLCD) may be described as a TLCDS connected to the structure with an actuator. The control of ATLCDs in wind excited towers has been studied by adopting a control algorithm that analyses the dynamic structural response [11]. The algorithm produces a feedback control force, which drives the TLCDS to counter the structural response. It was concluded that the ATLCD can significantly reduce the wind induced acceleration response of towers.

Over fifty years ago, it was discovered that magnetorheological (MR) suspensions possess the ability to reversibly change from free flowing, linear viscous liquids to semi-solids having a controllable yield strength within milliseconds of being exposed to a magnetic field [12]. MR fluids are suspensions of micron sized, magnetically polarizable particles in a carrier medium such as ethanol. The volume fraction of the particles is usually between 20% and 60%. The polarization induced in the suspended particles by the external magnetic field results in columnar structures forming in a direction parallel to the magnetic field. The application of MR fluids in civil engineering structures have been conducted by [13-18]. It was shown that a variable orifice within a TLCDS can greatly enhance the performance of the structure-TLCD

system [19]. The formation of columnar structures in the MRTLCD as a result of a magnetic field will act like an instantaneously variable orifice to an extent and restrict flow in the MRTLCD, allowing semi-active control of the structure-MRTLCD system.

When used in conjunction with TLCDs, the semi-active MRTLCD theoretically combines the benefits of active and passive control methods by using sensors to produce an optimum damping effect. A change in field applied to the horizontal tube in the MRTLCD imparts a controllable yield stress on the liquid flowing in the locality and hence allows for a degree of control of the MRTLCD. A semi-active optimal control method for non-linear multi-degree-of-freedom systems with MRTLCD under wind excitation, which combines the benefits of active and passive control methods was developed [20]. The semi-active MRTLCD installed at the top floor of a 50-storey building driven by a proposed optimal control strategy has been theoretically investigated [21]. Significant response reduction in terms of displacement, interstory drift and acceleration, in comparison with that obtained by using a passive TLCD, were recorded. The MRTLCDs investigated by [20-21] both utilize the passive damping properties of the residing MR fluid and assume a continuous power supply to allow the operation of the semi-active control methods developed. The passive damping properties of an MRTLCD installed on a SDOF structure were investigated [22]. It was found that with an appropriate MR fluid residing within the TLCD, reductions in the passive response of a SDOF structure with MRTLCD installed are similar to TLCDs with water when compared to the structure without MRTLCD. In times of power failure, it is possible for MRTLCDs to be relied upon to provide high levels of damping to the structural system. Whilst theoretical investigations into the operation of the MRTLCD have been undertaken, an experimental investigation into the implementation and potential real life use of an MRTLCD with applied field has not been undertaken.

This paper investigates the change in the damping response of structure-MRTLCD systems with applied magnetic field perpendicular with the direction of fluid flow. The MR fluid tested was a low viscosity MR fluid, custom designed by Liquids Research LTD (Wales). MRTLCDs with and without orifice in the horizontal section were constructed. The response of the structure-MRTLCD system to a sinusoidal base excitation and random excitation is investigated.

## 2 Experimental investigation

### 2.1 Model structure

A tall, slender aluminium annular pipe was used to model the tubular tower of a flexible structure and a pre-determined steel weight was placed at the top of the tube in order to deliver the required fundamental natural frequency. The lumped mass at the top of the structure consisted of metal strips with a thick, clear acrylic sheet placed under the MRTLCD. The underlying acrylic sheet guaranteed no secondary magnetic effects developing around the MRTLCD. The mass at the top of the structure, the Young's modulus, the second moment of area, the external diameter and the length of the tower were 6.2kg,  $6.9 \times 10^{10} \text{ Nm}^{-2}$ ,  $2.7 \times 10^{-9} \text{ m}^4$ , 0.02m and 1m, respectively. The model with attached MRTLCD can be seen in Fig. (1). A frequency generator was used as an input to the shaker in order to generate pre-defined excitations into the structure. Accelerometers were placed at the point of load application and at the peak of the structure, as been concluded that accelerometers provide the most effective response data for control of structures [11]. A force transducer was also placed at the point of excitation. The OROS data acquisition system was used to provide fast simultaneous data acquisition and digitisation of the multiple channel analog inputs used in the experiments. NVGate, the OROS noise and vibration software platform, controlled the analysis and measurements carried out by the OROS data acquisition system.



**Figure 1: Experimental structure-MRTLCD system**

From an applied initial peak displacement of 50mm, the fundamental natural frequency and equivalent damping ratio for the undamped structure were obtained. The equivalent viscous damping and natural frequency of the undamped structure obtained from the free vibration tests are 0.96 % and 9.486 rad/s, respectively. Tests with increasing applied excitation were performed to prove that the structure behaved linearly.

## **2.2 MRTLCD**

The MRTLCD was constructed using clear acrylic sheets and fastened permanently to the thick acrylic sheet at the top of the structure. The secure connection guaranteed the transfer of the shear force generated from the MRTLCD into the structure. A theoretical natural frequency of the structure without MRTLCD (9.466 radians per second) was obtained by modeling the structure without MRTLCD as a uniform beam with a lumped mass at the top. For the experimental tests undertaken on the structure, the MRTLCD was tuned to the natural frequency of the structure subject to the condition that the displacement of the liquid should be less than  $(L-B)/2$ . This condition ensures that the free surface of the liquid does not enter the horizontal pipe. Optimum tuning ratios for certain mass ratio cases and various natural damping values of the structure have been suggested [8-10]. The natural frequency of the MRTLCD is given by  $\omega_1 = \sqrt{(2g/L)}$  where  $L$  is the length of the liquid column and  $g$  is taken as 9.81 m/s/s. The MRTLCD had a 20mm square cross section and the horizontal section was 160mm. The tuning ratio and the ratio of the length of the horizontal part of the MRTLCD to the length of the liquid column ( $\alpha$ ) were 96.9% and 0.63, respectively. Two MRTLCDs, one without orifice and one with a 10mm square opening in the horizontal tube, were constructed. The coils of the MRTLCD with orifice were spaced 10mm closer together, therefore enabling a higher magnetic field to be applied to the MRTLCD.

## 2.3 Magnetic coils

To create an adequate magnetic flux in the vicinity of the flowing liquid, two magnetic coils on each side of the horizontal tube were used to provide an a sufficient magnetic flux density. The objective of the design of the magnetic coils was to guide and focus magnetic flux into a region of active magnetic fluid, e.g. the central region of the horizontal tube in the MRTLCD. The set up of the coils was such as to establish a transverse magneto-resistance to the flow of liquid. The coil set consisted of 4800 windings of a 0.3mm wire wrapped around a soft iron core of length 25mm and diameter 7mm. The resistance of the circuit was 101.8ohms. The coils were connected to two coupled power supplies that could input up to 70 volts into the magnetic circuit. The Tesla unit measures the concentration of a magnetic field, i.e. the number of field lines per square metre. The effective working range of the magnetic flux at the center point of the liquid in the MRTLCD without orifice, between the two coils, excluding losses in the liquid, was between 0 - 0.04 Tesla for the MRTLCD without orifice and 0 – 0.2 Tesla for the MRTLCD with orifice. These calculations are based on superposition, where the field at the center point between the coils is calculated assuming no magnetic losses to the fluid. The coils were mechanically connected to the MRTLCD so as to impose no additional damping in the structure. Free vibration tests confirmed the coils added no additional damping.

## 2.4 MR fluid

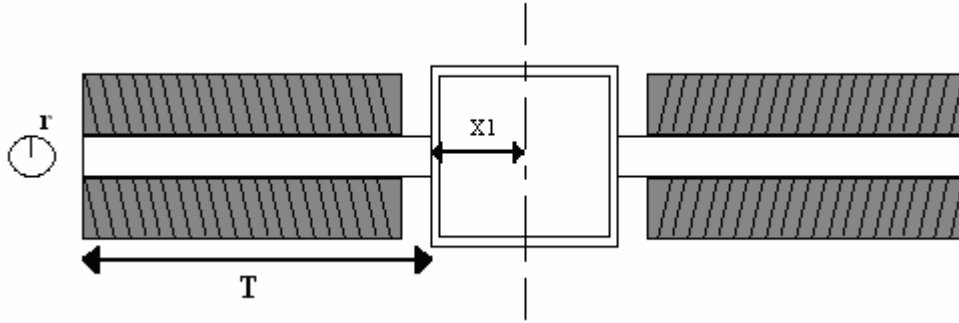
In most commercial MR fluids, a variety of proprietary additives are added to discourage gravitational settling and promote particle suspension, enhance lubricity and modify viscosity. As the viscosity and density of currently employed MR fluids are too high to use in practical applications of MRTLCDs, it was necessary to obtain an MR fluid with relatively low viscosity and density values. The ultimate strength of an MR fluid depends on the square of the saturation magnetization of the suspended particles, however the shear forces that the MR fluid is required to generate an orifice like effect in the MRTLCD in this paper are far lower than those required in a traditional MR fluid piston damper. The density of the custom MR fluid used in the testing was 2.49(g/ml). The MR fluid was 70% by weight magnetically active, less than 30% by weight carrier fluid and less than 30% by weight dispersants added.

# 3 Forced vibration tests

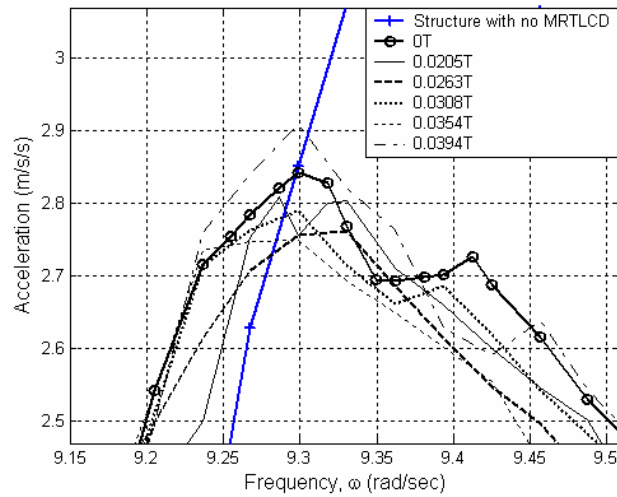
## 3.1 Influence of applied direct current on MRTLCD without orifice

The structure-MRTLCD system was first tested over a predefined frequency range with no magnetic field applied. At each frequency the acceleration and force measurements were taken when the structure-MRTLCD had reached a steady state response. The mass ratio,  $\mu$  (the ratio of the mass of the MRTLCD to the mass of the structure), was 3.96%. Different constant magnetic fields were applied to the MRTLCD while the structure-MRTLCD was sinusoidally excited over a frequency range, which encompassed the fundamental mode of vibration. In this section no orifice was used in the horizontal part of the MRTLCD. A schematic cross section of the application of the magnetic field in this section is displayed in Fig. (2). The value of  $X1$ ,  $T$  and  $r$  were 29mm, 25mm and 7mm, respectively. Fig. (3) shows the acceleration response of the undamped structure and the structure-MRTLCD due to various magnetic fields applied. The excitation force, which was kept constant throughout the various tests, was applied 0.33m from the base of the 1m structure. In the legend, the  $T$  refers to Tesla and  $0T$  refers to the structure-MRTLCD system with no field applied. As the variation of the response of the damped structure with magnetic field is primarily of interest, the peak structure without MRTLCD response is not included. The response of the structure with MRTLCD and no magnetic field applied is 25% less than the response of the structure without MRTLCD (3.8 m/s/s). There are changes in the structure-MRTLCD response when the magnetic

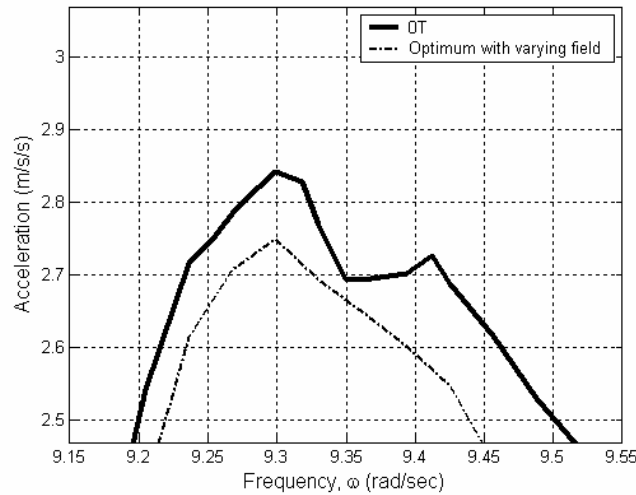
field is applied, however when no orifice is present these changes are not substantial. The main effect of the magnetic field in this case is to alter the location of the double peaked responses of the structure-MRTLCD in the frequency domain. With the optimal variation of the magnetic field, the structure-MRTLCD response is displayed in Fig. (4). The total response of the structure-MRTLCD with optimally applied magnetic fields is 27.6%, which represents only a small improvement in performance from the case where no magnetic field was applied.



**Figure 2: Schematic of the cross section of the horizontal section of the MRTLCD without orifice**



**Figure 3: Forced vibration response**



**Figure 4: Forced vibration response**

### 3.2 Influence of applied direct current on MRTLCD with orifice

The same steps as indicated in section 3.1 were repeated for the MRTLCD with orifice. A schematic cross section of the application of the magnetic field in this section is displayed in Fig. (5). The value of  $X_2$  was 17mm. Fig. (6) plots the response of the undamped structure and the structure-MRTLCD with various magnetic fields applied. When the MRTLCD is used without an applied magnetic field the response of the system is 29% less than that of the undamped structure. It can be seen that the response of the system is reduced with increasing magnetic field until 0.0509 Tesla is applied. With subsequent increases in the magnetic field, the system response is gradually increased from the level attained with 0.0509 Tesla. Subsequent tests were performed at smaller magnetic flux intervals around 0.0509 Tesla and the three best results can be seen in Fig. (7). Again, the double peaked response of the system is varied along the frequency domain. The total response reduction at resonance implementing a magnetic field of 0.0695 Tesla is 35.7%.

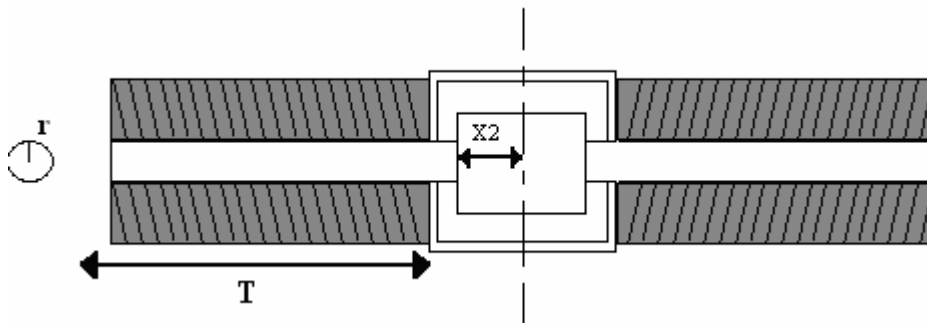


Figure 5: Schematic of the cross section of the horizontal section of the MRTLCD with orifice

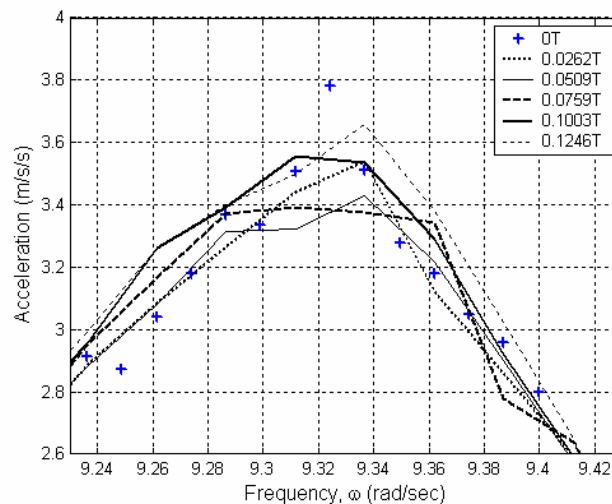
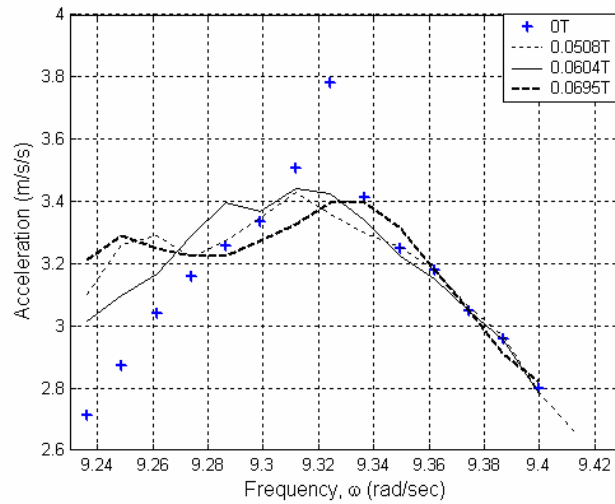


Figure 6: Forced vibration response

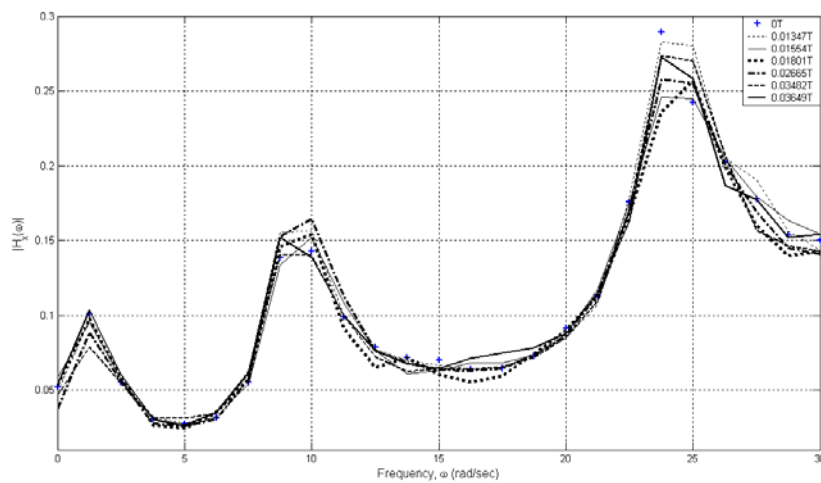


**Figure 7: Forced vibration response**

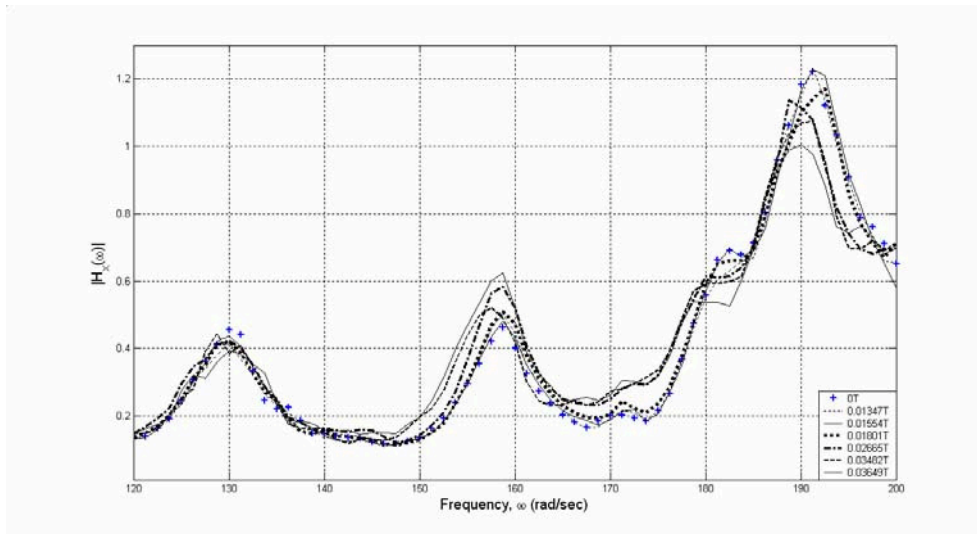
### 3.3 Response of MRTLCD to white noise excitation

#### 3.3.1 White noise excitation applied at 0.33m from the base

A white noise excitation was applied to the structure at a distance of 0.33m from the base of the structure. The frequency range of the white noise excitation applied to the structure was from 0-1000 rad/sec. A white noise excitation was applied to the structure-MRTLCD for 14 different cases of applied magnetic field up to the point where there was no visible movement of the MR fluid in the MRTLCD. Figs. (8)-(10) display various response peaks in the frequency response spectrum using six representative magnetic fields from the original 14. The transfer ratio,  $|H_x(\omega)|$ , which is the FFT of the tip acceleration divided by the FFT of the acceleration at the loading point, is plotted on the y-axis.

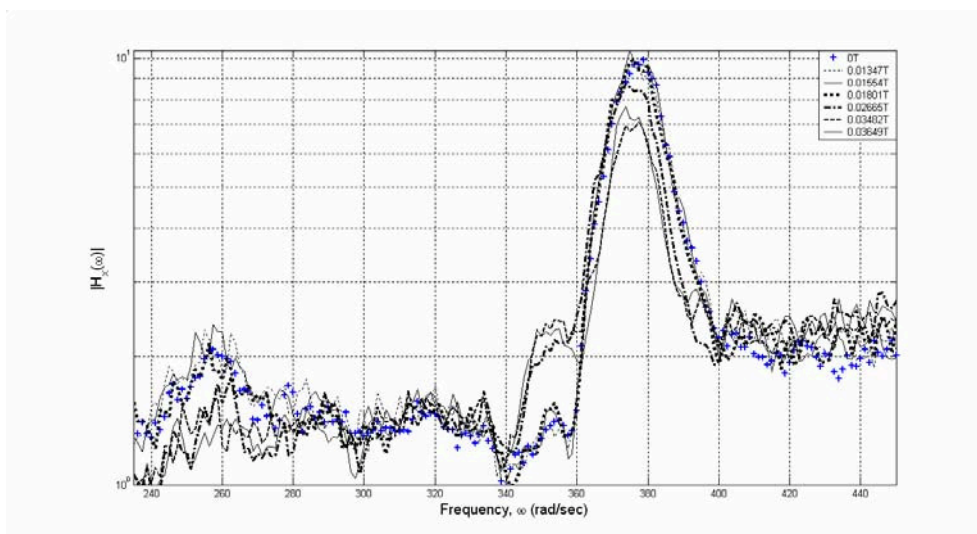


**Figure 8: Forced vibration response-frequency range 0-30 rad/sec**



**Figure 9: Forced vibration response-frequency range 120-200 rad/sec**

Variation of the magnetic field in the horizontal pipe of the MRTLCD had an effect on the structure-MRTLCD across all the excited modes in the frequency spectrum. In Fig. 8 at the peaks at 1.25 rad/sec, 9.85 rad/sec and 23.8 rad/sec, the reduction from the response of the structure-MRTLCD system with 0 Tesla is 23%, 4% and 19%, respectively. In Fig. 9 at the peaks at 130 rad/sec, 159 rad/sec and 192 rad/sec, the reduction from the response of the structure-MRTLCD system with 0 Tesla applied is 15%, 0% and 21%, respectively. Fig. 10 is plotted in log scale as the peak response of the structure-MRTLCD system is contained within. In Fig. 10 at the peaks at 1.25 rad/sec, 9.85 rad/sec and 23.8 rad/sec, the reduction from the response of the structure-MRTLCD system with 0 Tesla is 38% and 32%, respectively. Thus, the non-linear effect of the MRTLCD has a greater effect on the broadbanded response than that of the sinusoidal excitation and the effect of varying the magnetic field on the MRTLCD influences the structure-MRTLCD response across all frequencies. The root-mean-square values of the transfer ratio for each of the tests are given in table 1. There is a significant reduction in the response from the structure without MRTLCD for all the cases where there is an MRTLCD with current. On average across all the frequencies, the case without applied field is the most efficient, however additional performance can be gained by varying the applied field to an optimum value when specific modes are excited.



**Figure 10: Forced vibration response-frequency range 235-450 rad/sec**

### 3.3.2 White noise excitation applied at 0.66m from the base

A white noise excitation was next applied to the structure at a distance of 0.66m from the base of the structure. The results can be viewed in Figs. (11)-(13). Reductions in the response of the structure of 53% and 36% are evident in the two large peak responses at 158rad/s and 378 rad/s.

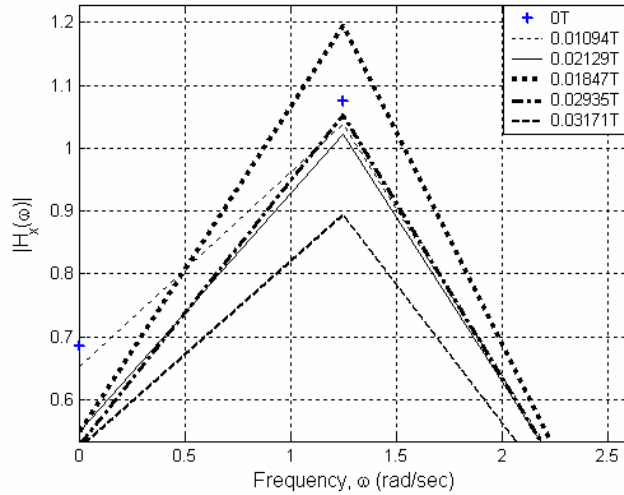


Figure 11: Response of upper load excitation

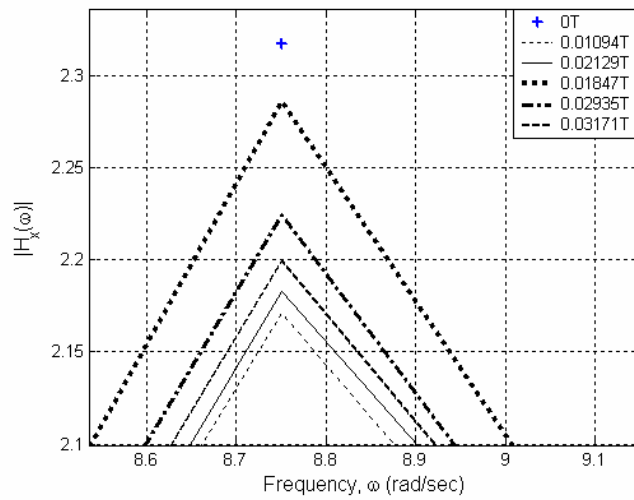
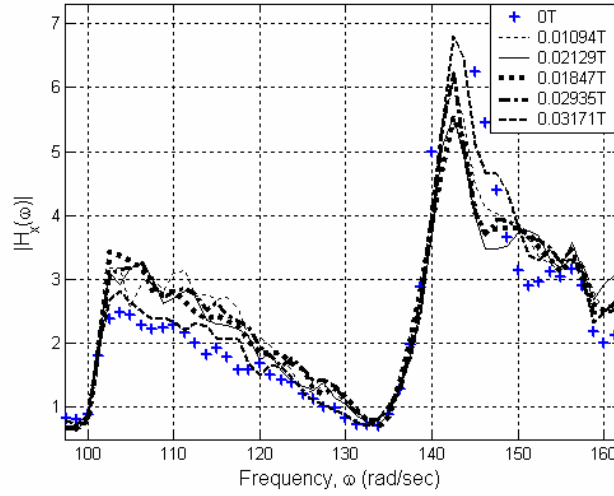


Figure 12: Acceleration response for first peak



**Figure 13: Acceleration response for first peak**

Variation of the magnetic field in the horizontal pipe of the MRTLCD had an effect on the structure-MRTLCD across all the excited modes in the frequency spectrum. In Fig. 11, which shows the first peak in the frequency response spectrum, the reduction from the response of the structure-MRTLCD system with 0 Tesla is 17.5%. In Fig. 12 at the next peak in the frequency response spectrum, the reduction from the response of the structure-MRTLCD system with 0 Tesla is 5%. In Fig. 13 at the peaks at approximately 104 rad/sec and 146 rad/sec, the reduction from the response of the structure-MRTLCD system with 0 Tesla is 0% and 22%, respectively. Thus, by varying the applied magnetic field from 0T to 0.03171T (which is around the field needed to inhibit any visible motion of the MR fluid within the MRTLCD) additional performance gains can be obtained by using the MRTLCD. The root-mean-square values of the transfer ratio for sections are given in table 1. In this case, an applied field of  $31.7 \times 10^{-3}$  is the most efficient across all frequencies.

## 4 Conclusion

In this paper an attempt has been made to introduce and demonstrate the practicality of the MRTLCD. An experimental apparatus was constructed which was capable of inputting predefined excitations at specific amplitudes and frequencies to any point along the model structure. An MRTLCD was designed and constructed such that two solenoid coils could produce a magnetic field in a transverse direction to the fluid flow. In all structure-MRTLCD cases, the structural response was greatly decreased compared to that of the structure without MRTLCD. It was observed that additional performance can be gained by varying the applied field to an optimum value when specific modes are excited. The influence of the magnetic field was found to be greater for broad-banded excitations than for sinusoidal excitations. When slender structures are excited by a wind excitation, MRTLCDs can be used to effectively dampen excited modes across all frequencies. The forced vibration response tests indicate that under a semi-active control strategy, MRTLCDs may be practically implemented to reduce induced high amplitude structural vibrations in structures in a manner where the control forces are applied instantaneously at low magnetic fields.

<b>RMS of <math> H_x(\omega) </math> for 0.33m excitation from base</b>	<b>Structure without MRTLCD</b>	<b>0T</b>	<b>4.7T</b> $10^{-3}$	<b>6.9T</b> $10^{-3}$	<b>10.3T</b> $10^{-3}$	<b>13.4T</b> $10^{-3}$	<b>15.5T</b> $10^{-3}$	<b>18.0T</b> $10^{-3}$	<b>20.7T</b> $10^{-3}$	<b>22.6T</b> $10^{-3}$	<b>26.6T</b> $10^{-3}$	
		3.108	1.9530	2.0592	2.1188	1.9530	2.1019	2.0826	2.0740	2.1110	2.1221	2.0946
			<b>29.7T</b> $10^{-3}$	<b>31.2T</b> $10^{-3}$	<b>34.8T</b> $10^{-3}$	<b>36.4</b> $10^{-3}$						
		2.0892	2.1485	2.0054	2.0790							
<b>RMS of <math> H_x(\omega) </math> for 0.66m excitation from base</b>	<b>Structure without MRTLCD</b>	<b>0T</b>	<b>1.84T</b> $10^{-3}$	<b>4.09T</b> $10^{-3}$	<b>7.88T</b> $10^{-3}$	<b>10.94T</b> $10^{-3}$	<b>13.4T</b> $10^{-3}$	<b>15.5T</b> $10^{-3}$	<b>18.4T</b> $10^{-3}$	<b>21.29T</b> $10^{-3}$	<b>25.1T</b> $10^{-3}$	
		12.98	8.9542	8.3606	8.5825	8.6707	8.7465	8.6757	8.5029	8.5638	8.4720	8.4397
			<b>27.0T</b> $10^{-3}$	<b>29.3T</b> $10^{-3}$	<b>31.7T</b> $10^{-3}$	<b>34.59</b> $10^{-3}$	<b>37.47</b> $10^{-3}$					
		8.6973	8.7079	7.7843	8.5336	8.6845						

**Table 1: RMS values**

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